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# Tests for General Error Specifications and Non-Nested Models: A Simultaneous Approach

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## ABSTRACT

This paper is concerned with joint test of non-nested models and simultaneous departures from homoskedasticity, serial independence and normality of the disturbance terms. Locally equivalent alternative models are used to construct joint tests since they provide a convenient way to incorporate more than one type of departures from the classical conditions. the joint tests represent a simple asymptotic solution to the "pre-testing" problem in the context of non-nested linear regression models. Our simulation results indicate that the proposed tests have good finite sample properties.

**Key Words and Phrases:** locally equivalent alternative models; non-normal errors; non-spherical errors; pre-testing problem; simulation study



## 1. INTRODUCTION

In recent years a substantial literature has been developed for testing non-nested regression models. While the available procedures are now frequently used for both testing and modelling purposes, in many cases it would seem that the non-nested models are presumed to have disturbances satisfying the classical conditions of serial independence (I), homoskedasticity (H) and normality (N). In practice, while departures from the classical conditions occur quite frequently, it is not straightforward to modify the available test procedures to incorporate all the departures, especially non-normality ( $\bar{N}$ ) of the disturbances. Moreover, in the nested testing situation, most of the popular tests are “one-directional” in that they are designed to test against only a single alternative hypothesis, and in most cases the tests are valid only when the other standard assumptions are satisfied. Many researchers have found that the one-directional tests are not robust in the presence of other misspecifications [see Bera and Jarque (1982) and the references cited therein]. In Section 2, the robustness of several well known tests for both nested and non-nested hypotheses is discussed briefly. In Section 3, we develop a procedure for testing non-nested models together with simultaneously checking the sphericity and normality of the disturbance terms. Locally equivalent alternative models are used to construct joint tests since they provide a convenient method for incorporating more than one type of departures from the classical conditions. The joint tests represent a simple asymptotic solution to the “pre-testing” problem in the context of non-nested linear regression models. In Section 4, we investigate the finite sample properties of our proposed tests through a limited simulation study. Some concluding remarks are given in Section 5.

## 2. ROBUSTNESS OF SEVERAL EXISTING TESTS

In testing nested hypotheses, three kinds of situations can occur, namely undertesting, overtesting and complete misspecification. When departures

from the null hypothesis are multi-directional and an one-directional test is used, undertesting is said to occur. On the other hand, in overtesting, the test statistic overstates the alternative hypothesis. Complete misspecification occurs when the assumed alternatives and the data generating process (DGP) are mutually exclusive. In both undertesting and overtesting, a loss of power is to be expected. However, joint testing of several hypotheses is rarely performed in practice, so that inferences are generally affected by undertesting. By drawing upon some Monte Carlo results from Bera (1982) and Bera and Jarque (1982), we highlight the effects of undertesting and the subsequent non-robustness of a commonly used test of heteroskedasticity. For an analytic study on the non-robustness of specification tests under different kinds of misspecifications, see Bera and Yoon (1990).

Three convenient simplifying assumptions are usually made in standard regression analysis, namely H, I and N. In what follows, we consider the Breusch and Pagan (1979) Lagrange multiplier (LM) test for heteroskedasticity ( $\bar{H}$ ). The data are generated under different combinations of  $\bar{N}$  (the  $t$  distribution with five degrees of freedom),  $\bar{H}$  (additive heteroskedasticity) and serial dependence  $\bar{I}$  (first-order serial correlation); for further details, see Bera (1982) and Bera and Jarque (1982). The estimated powers of the test statistic under different DGPs are given in Table 1. The results are for sample size 50 and significance level .10 and are based on 500 replications. Therefore, the maximum standard errors of the reported numbers would be  $\sqrt{.5(1 - .5)/500} \simeq .022$ .

TABLE 1. Robustness of the LM test for heteroskedasticity

|                 | <u>Nature of Alternative Model</u> |                         |                         |                         |                         |                         |
|-----------------|------------------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
|                 | <u>Correct</u>                     | <u>Contaminated</u>     |                         |                         | <u>Misspecified</u>     |                         |
| DGP             | $\bar{H}\bar{I}\bar{N}$            | $\bar{H}\bar{I}\bar{N}$ | $\bar{H}\bar{I}\bar{N}$ | $\bar{H}\bar{I}\bar{N}$ | $\bar{H}\bar{I}\bar{N}$ | $\bar{H}\bar{I}\bar{N}$ |
| Estimated power | .726                               | .660                    | .270                    | .302                    | .084                    | .222 .128               |

When the DGP contains only heteroskedasticity (that is,  $\overline{HIN}$ ), the LM test is asymptotically optimal against the correct alternative and power is estimated to be .726. However, the estimated powers fall to .660 and .270 when the data are contaminated by the  $t$  distribution ( $\overline{HIN}$ ) and first-order serial correlation ( $\overline{HIN}$ ), respectively. The effect of  $\bar{I}$  on the LM test of  $\overline{H}$  is quite substantial. When both  $\bar{I}$  and  $\bar{N}$  contaminate the DGP, the estimated power is only .302.

The final three cases are completely misspecified in that the LM test is seeking to detect  $\overline{H}$  when  $\overline{H}$  does not exist but,  $\bar{I}$ ,  $\bar{N}$  or both  $\bar{I}$  and  $\bar{N}$  are present. The interpretation of the estimated powers depends on how the tests are to be viewed. If the test is seen exclusively as a test of  $\overline{H}$ , then the estimated powers should equal the estimated significance levels or sizes. It is clear from Table 1 that the estimated sizes are significantly greater than the nominal size of the test in all three cases. However, if the test is used purely as a significance test, the estimated powers seem to be quite low, and are much lower than those for the case where the alternative model is contaminated. Regardless of the interpretation, therefore, the test does not perform well. The one-directional test is not robust when the alternative model is contaminated and the test is not satisfactory in the completely misspecified case, regardless of how it is interpreted. Here we have used Breusch and Pagan (1979) test for  $\overline{H}$  for illustration only. As suggested by Payen (1980) and Koenker (1981), this test can be made robust by studentizing it. Bera and Jarque (1981) report some Monte Carlo results on the performance of this robustified test.

Turning now to the case of non-nested hypotheses, extensive Monte Carlo experiments have been conducted by Godfrey and Pesaran (1983) and Godfrey et al. (1988). The first of these two papers is concerned with the selection of regressors in two non-nested linear regression models, and examines the Cox test of Pesaran (1974), two mean- and variance-adjusted versions of the Cox test, the J test of Davidson and MacKinnon (1981), the JA test of Fisher and McAleer (1981), and the standard  $F$  test applied to the



comprehensive model constructed as a union of the two models. Since the tests are valid only asymptotically when the disturbances are not normally distributed, Godfrey and Pesaran (1983) examine the robustness of the tests to errors drawn from the log-normal distribution and the chi-squared distribution with two degrees of freedom. Their experiments indicate that the finite sample significance levels are not significantly distorted and are broadly similar to those for the case of normally distributed errors. Although estimated powers tend to be greater when the errors are drawn from the two non-normal distributions compared with the normal case, the relative rankings of the tests in terms of power are not affected by the non-normality.

Various procedures are considered by Godfrey et al. (1988) for testing the non-nested linear and logarithmic functional forms. The test procedures are classified as non-nested tests, two versions of the LM test and a variable addition test based on the more general Box-Cox transformation, and diagnostic tests of (possible) functional form misspecification against an unspecified alternative hypothesis. If the logarithmic model is to be taken seriously, the dependent variable of the linear model cannot take on negative values and the disturbances of the linear model cannot be normal. Therefore, it is essential to examine the robustness of the tests to non-normality of the errors, even if the primary consideration rests with testing the non-nested functional forms. Godfrey et al. (1988) examine the finite sample significance levels and powers of the tests when the disturbances follow the gamma (2,1) distribution, the log-normal distribution and the  $t$  distribution with five degrees of freedom. The two versions of the LM test based on the Box-Cox model are found to be highly sensitive to non-normality in that the estimated significance levels are far greater than those predicted by asymptotic theory, even when the sample size is eighty. On the other hand, the variable addition tests in the three categories are found to be robust to non-normality of the errors, and their relative rankings in terms of power are not affected by departures from normality.

### 3. JOINT TESTS

The standard situation for testing non-nested linear regression models with normal and spherical errors is as follows. It is desired to test the null model  $H_0$  against the non-nested alternative  $H_1$ , where the two models are given as

$$H_0 : y = X\beta + u_0, \quad u_0 \sim \mathcal{N}(0, \sigma_0^2 I_n)$$

and

$$H_1 : y = Z\gamma + u_1, \quad u_1 \sim \mathcal{N}(0, \sigma_1^2 I_n),$$

in which  $y$  is the  $n \times 1$  vector of observations on the dependent variable,  $X$  and  $Z$  are  $n \times k$  and  $n \times g$  matrices of observations on  $k$  and  $g$  linearly independent regressors,  $\beta$  and  $\gamma$  are  $k \times 1$  and  $g \times 1$  vectors of unknown parameters, and  $u_0$  and  $u_1$  are vectors of normally, independently and identically distributed disturbances. It is also assumed that  $X$  and  $Z$  are not orthogonal, and that the limits of  $n^{-1}X'X$ ,  $n^{-1}Z'Z$  and  $n^{-1}X'Z$  exist, with the first two positive definite and the third non-zero. If  $X$  and  $Z$  contain stochastic rather than fixed elements, the probability limits of the appropriate matrices must exist, and  $X$  and  $Z$  must be distributed independently of  $u_0$  and  $u_1$  under  $H_0$  and  $H_1$ , respectively.

In considering the consequences of testing for certain departures from sphericity, it will be convenient to rewrite the two models as

$$H_0 : y_t = x_t' \beta + u_{0t} \tag{1}$$

and

$$H_1 : y_t = z_t' \gamma + u_{1t}, \tag{2}$$

in which  $x_t'$  and  $z_t'$  are the  $t$ 'th rows of  $X$  and  $Z$ , respectively, and  $t = 1, 2, \dots, n$ . When the assumptions regarding  $u_{0t}$  and  $u_{1t}$  are not satisfied, some of the properties of the tests will be affected. For example, Pesaran

(1974) derived a test of non-nested linear regression models where the disturbances of each model follow a first-order autoregressive scheme. However, Pesaran's test is very complicated to apply in practice and a simple procedure is given in McAleer, Pesaran and Bera (1990). The effect of heteroskedasticity will be similar. In practice, a straightforward application of the tests suggested in Davidson and MacKinnon (1981) and Fisher and McAleer (1981) will not be valid since the standard errors will not be correct. However, use of a heteroskedasticity-consistent covariance matrix estimator will circumvent this problem. When the errors are not normal, Pesaran's test based on the work of Cox (1961, 1962) is still valid asymptotically, although its small sample properties will be affected. Normality is required for the test suggested by Fisher and McAleer (1981) to have the exact  $t$ -distribution under the null hypothesis; if normality does not hold, the test will be valid only asymptotically.

In the light of the above discussion, a basic requirement for applying standard non-nested testing procedures to achieve high power is that the models under consideration be well-specified. This means that tests for normality and sphericity, for example, are to be performed prior to testing the non-nested models themselves. An important, and frequently overlooked, aspect of testing non-nested models in this two-step procedure is the effect that such "pre-testing" may have on the levels of significance and powers of the non-nested tests. Therefore, it may be desirable to test the non-nested specifications jointly with departures from the classical assumptions regarding the disturbance terms. Such procedures will be particularly useful when there is a possibility of a non-normal disturbance term of unknown type, since it is frequently difficult to take account of general forms of non-normality in a straightforward manner. Joint tests may be constructed in a straightforward way by developing an approximate model which incorporates the various departures from the classical conditions into the systematic part of the model, so that the disturbances of the approximate model are normally,

independently and identically distributed. The advantage of joint tests over the two-step testing procedure lies in the way the joint testing procedure deals with the “pre-testing” problem, at least asymptotically. In the following section we will study the finite sample properties of a joint test through Monte Carlo experiments.

In the context of deriving LM diagnostic tests, Godfrey and Wickens (1982) suggested a way of obtaining a local approximation to a given model with a non-standard disturbance structure. Such approximations are called “locally equivalent alternative”(LEA) models. As an illustration, consider  $H_0$  in (1), where it is now *assumed* that the disturbance  $u_{0t}$  is given by

$$u_{0t} = \rho_0 u_{0t-1} + \epsilon_{0t}, \quad \epsilon_{0t} \sim \mathcal{NID}(0, \sigma_0^2), \quad |\rho_0| < 1 \quad (3)$$

for  $t = 2, 3, \dots, n$ . A LEA model to  $H_0$  may be written as

$$H_0^* : y_t = x_t' \beta + \rho_0 \tilde{u}_{0t-1} + \epsilon_{0t}, \quad (4)$$

in which  $\tilde{u}_{0t} = y_t - x_t' \tilde{\beta}$  and  $\tilde{\beta}$  is the ordinary least squares estimate of  $\beta$  under  $H_0$ . The models  $H_0^*$  in (4) and  $H_0$  in (1) and (3) are “equivalent” in the sense that:

- (i) when  $\rho_0 = 0$ ,  $H_0$  and  $H_0^*$  are identical;
- (ii) when  $\rho_0 = 0$ , then  $\partial l_t(\tilde{\beta}, \tilde{\sigma}_0^2, \rho_0) / \partial \rho_0 = \partial l_t^*(\tilde{\beta}, \tilde{\sigma}_0^2, \rho_0) / \partial \rho_0$ ,  
where  $l_t(\beta, \sigma^2, \rho_0)$  and  $l_t^*(\beta, \sigma^2, \rho_0)$  are the log-density functions for the  $t$ 'th observation under  $H_0$  and  $H_0^*$ , respectively.

Godfrey (1981) has shown that, in testing  $\rho_0 = 0$  for local alternatives, the likelihood ratio test of  $\rho_0 = 0$  applied to  $H_0$  in (1) and (3) and the LM test of  $\rho_0 = 0$  applied to  $H_0^*$  in (4) have similar power. Thus, for values of  $\rho_0$  in the neighbourhood of zero,  $H_0$  and  $H_0^*$  may be regarded as equivalent.

Now let us consider (1) and (2) allowing for the possibility that the disturbances  $u_{it}$  ( $i = 0, 1$ ) follow stationary autoregressive processes of order

$p_i(i = 0, 1)$ , namely  $\text{AR}(p_i)$ , as follows:

$$u_{it} = \sum_{j=1}^{p_i} \rho_{ij} u_{it-j} + \epsilon_{it}, \quad i = 0, 1$$

where  $t = p + 1, p + 2, \dots, n$  and  $p = \max(p_0, p_1)$ . In this case, a locally equivalent form of (1) may be written as

$$H_0^* : y_t = x_t' \beta + \sum_{j=1}^{p_0} \rho_{0j} \tilde{u}_{0t-j} + \epsilon_{0t},$$

where  $\tilde{u}_{0t} = y_t - x_t' \tilde{\beta}$ , as before.

Although several procedures are available in the literature [for a recent review, see McAleer and Pesaran (1986)], a convenient test of the null model against both the non-nested alternative  $H_1$  and  $\text{AR}(p_0)$  disturbances can be performed by testing  $\alpha = \rho_{01} = \rho_{02} = \dots = \rho_{0p_0} = 0$  in the auxiliary linear regression required for the J test of Davidson and MacKinnon (1981), namely

$$y_t = x_t' \beta + \sum_{j=1}^{p_0} \rho_{0j} \tilde{u}_{0t-j} + \alpha \hat{y}_{1t} + \epsilon_t,$$

where  $\hat{y}_{1t}$  is the predicted value of  $y_t$  from (2), namely

$$\hat{y}_{1t} = z_t' \hat{\gamma} + \sum_{j=1}^{p_1} \hat{\rho}_{1j} \hat{u}_{1t-j},$$

$\hat{u}_{1t} = y_t - z_t' \hat{\gamma}$  and  $\hat{\gamma}$  is the maximum likelihood estimate of  $\gamma$  under  $H_1$ .

An attractive feature of this approach is that other departures from the classical conditions may be handled in an equally straightforward manner. Consider the following general form of the distribution of the disturbance term for  $H_0$  of (1), where  $u_{0t}$  follows an autoregressive process of order  $p_0$ , namely

$$u_{0t} = \sum_{j=1}^{p_0} \rho_{0j} u_{0t-j} + \epsilon_{0t},$$



and  $\epsilon_{0t}$  is independently distributed. The density of  $\epsilon_{0t}$ , denoted by  $g(\epsilon_{0t})$ , is assumed to be a member of the symmetric Pearson family of distributions. This is not a very restrictive assumption since this family encompasses many distributions such as the normal, Student  $t$  and  $F$ . The density of  $\epsilon_{0t}$  is then given by

$$g(\epsilon_{0t}) = \exp[\Psi(\epsilon_{0t})] \left[ \int_{-\infty}^{\infty} \exp[\Psi(\epsilon_{0t})] d\epsilon_{0t} \right]^{-1}, \quad -\infty < \epsilon_{0t} < \infty$$

where

$$\Psi(\epsilon_{0t}) = \int [-\epsilon_{0t}/(c_{0t} + c_1\epsilon_{0t}^2)] d\epsilon_{0t}.$$

When  $c_1 = 0$ ,  $g(\epsilon_{0t})$  reduces to a normal density with mean zero and variance  $c_{0t}$ . Heteroskedasticity is introduced through  $c_{0t}$ . It is assumed that

$$c_{0t} = h \left( \sum_{l=1}^{q_0} \phi_l v_{lt} \right),$$

where the elements of the  $q_0 \times 1$  vector  $v_t = (v_{1t}, v_{2t}, \dots, v_{q_0t})'$  are fixed and measured around their means,  $h(\cdot)$  is a twice differentiable function with  $h(0) = \sigma_0^2$ , and  $\phi_l (l = 1, 2, \dots, q_0)$  are unknown parameters. Under these circumstances, the disturbances for  $H_0$  in (1) are now *non-normal, heteroskedastic and serially dependent*. A simple local approximation to this complicated model may be written as

$$H_0^{**} : y_t = x_t' \beta + \sum_{j=1}^{p_0} \rho_{0j} \tilde{u}_{0t-j} + \tilde{u}_{0t} \sum_{l=1}^{q_0} \phi_l v_{lt} + c_1 r_t + \epsilon_{0t}, \quad (5)$$

in which

$$r_t = (\tilde{u}_{0t}^3 - 3\tilde{u}_{0t}\tilde{\sigma}_0^2)/(4\tilde{\sigma}_0^2),$$

$$\tilde{\sigma}_0^2 = n^{-1} \sum_{t=1}^n \tilde{u}_{0t}^2,$$

with  $\epsilon_{0t} \sim \mathcal{NID}(0, \sigma_0^2)$  for all  $t$ .

To verify whether model (5) is a LEA model, we first note that, when  $\rho_0 = (\rho_{01}, \rho_{02}, \dots, \rho_{0p_0})' = 0$ ,  $\phi = (\phi_1, \phi_2, \dots, \phi_{q_0})' = 0$  and  $c_1 = c_2 = 0$ ,  $H_0$  in (1) and  $H_0^{**}$  in (5) are identical and  $\epsilon_{0t} = u_{0t}$ , so that condition (i) is satisfied. Godfrey and Wickens (1982) verified condition (ii) above for the case of serial correlation and heteroskedasticity. It is, therefore, necessary to consider only the non-normal component. First, it can be shown that there is no contribution from the Jacobian term. The Jacobian from  $c_1 r_t$  is asymptotically given by [see Godfrey and Wickens (1982, p.85)]

$$\sum_t \ln \left[ 1 - c_1 \frac{3(u_{0t}^2 - \sigma_0^2)}{4\sigma_0^4} \right]$$

which, under local alternatives, reduces to

$$-\frac{3c_1}{4\sigma_0^4} \sum_t (u_{0t}^2 - \sigma_0^2),$$

which is  $O_p(1)$ . Therefore, for the purpose of developing a joint test, we can ignore the Jacobian term.

It can also be shown that the score with respect to  $c_1$  is the same under  $H_0$  and  $H_0^{**}$ . From Bera and Jarque (1982, p.78), it follows that

$$\frac{\partial l_t(\tilde{\beta}, \tilde{\sigma}_0^2, 0)}{\partial c_1} = \frac{\tilde{u}_{0t}^4}{4\tilde{\sigma}_0^4} - \frac{3}{4}$$

where  $l_t(\beta, \sigma_0^2, \gamma)$  is the log-density function under  $H_0$ , with  $\gamma = (\rho_{01}, \rho_{02}, \dots, \rho_{0p_0}, \phi_1, \phi_2, \dots, \phi_{q_0}, c_1)'$ . Using the information that  $\epsilon_{0t} \sim \mathcal{NID}(0, \sigma_0^2)$  for all  $t$ , it follows from (5) that

$$\frac{\partial l_t^{**}(\tilde{\beta}, \tilde{\sigma}_0^2, 0)}{\partial c_1} = r_t \tilde{u}_{0t} / \tilde{\sigma}_0^2,$$

where  $l_t^{**}(\cdot)$  is the log-density function under  $H_0^{**}$ . Since

$$\sum_{t=1}^n \frac{\partial l_t(\tilde{\beta}, \tilde{\sigma}_0^2, 0)}{\partial c_1} = \sum_{t=1}^n \frac{\partial l_t^{**}(\tilde{\beta}, \tilde{\sigma}_0^2, 0)}{\partial c_1},$$

the score under the original and LEA models is the same. One component of the LM test for normality is based on this score value [see Bera and Jarque (1981), and Jarque and Bera (1987)].

If it were desired to test serial independence, homoskedasticity and normality under  $H_0$ , we would test the parameter restrictions

$$\rho_0 = 0, \phi = 0, c_1 = 0$$

in equation (5). This joint test procedure has been suggested by Bera and Jarque (1982). However, they did not consider the possibility of the non-nested alternative  $H_1$  together with the non-sphericity and non-normality of the disturbance term under  $H_0$ .

If suitable predictions  $\hat{y}_{1t}$  from a non-nested alternative  $H_1$  are augmented to equation (5) to yield the auxiliary regression given by

$$y_t = x_t' \beta + \sum_{j=1}^{p_0} \rho_{0j} \tilde{u}_{0t-j} + \tilde{u}_{0t} \sum_{l=1}^{q_0} \phi_l v_{lt} + c_1 r_t + \alpha \hat{y}_{1t} + \epsilon_{0t}, \quad (6)$$

then the null hypothesis, namely equation (1) with  $u_{0t} = \epsilon_{0t}$ , involves a joint test of

$$H : \rho_0 = 0, \phi = 0, c_1 = 0, \alpha = 0. \quad (7)$$

This joint hypothesis can be tested by applying the LM procedure, for example, directly to equation (6) or by using an appropriately adjusted  $F$  test [see Godfrey and Wickens (1982)]. In computing the LM test, the most convenient form is  $nR^2$ , that is, the sample size times the (uncentered) coefficient of determination in the auxiliary regression of a vector of ones on the

following variables:

$$\begin{aligned}\frac{\partial l_t^{**}}{\partial \beta} &= x_t' \tilde{u}_{0t} / \tilde{\sigma}_0^2 \\ \frac{\partial l_t^{**}}{\partial \sigma_0^2} &= (\tilde{u}_{0t}^2 - \tilde{\sigma}_0^2) / (2\tilde{\sigma}_0^4) \\ \frac{\partial l_t^{**}}{\partial \rho_{0j}} &= \tilde{u}_{0t} \tilde{u}_{0t-j} / \tilde{\sigma}_0^2, \quad j = 1, 2, \dots, p_0 \\ \frac{\partial l_t^{**}}{\partial \phi_l} &= (\tilde{u}_{0t}^2 v_{lt} / \tilde{\sigma}_0^2) - v_{lt}, \quad l = 1, 2, \dots, q_0 \\ \frac{\partial l_t^{**}}{\partial c_1} &= (r_t \tilde{u}_{0t} / \tilde{\sigma}_0^2) - 3(\tilde{u}_{0t}^2 - \tilde{\sigma}_0^2) / (4\tilde{\sigma}_0^4)\end{aligned}$$

and

$$\frac{\partial l_t^{**}}{\partial \alpha} = \hat{y}_{1t} \tilde{u}_{0t} / \tilde{\sigma}_0^2.$$

The set of regressors given above can be simplified to

$$\begin{aligned}(x_t' \tilde{u}_{0t}, \tilde{u}_{0t} - \tilde{\sigma}_0^2, \tilde{u}_{0t} \tilde{u}_{0t-1}, \dots, \tilde{u}_{0t} \tilde{u}_{0t-p_0}, (\tilde{u}_{0t}^2 - \tilde{\sigma}_0^2) v_{1t}, \dots, (\tilde{u}_{0t}^2 - \tilde{\sigma}_0^2) v_{q_0 t}, \\ 4r_t \tilde{u}_{0t} \tilde{\sigma}_0^2 - 3(\tilde{u}_{0t}^2 - \tilde{\sigma}_0^2), \hat{y}_{1t} \tilde{u}_{0t}).\end{aligned}$$

Although this  $nR^2$  version of LM test is very convenient to use, it often leads to statistics that behave badly in finite samples [see Davidson and MacKinnon (1983) and Bera and McKenzie (1986)].

As noted by Godfrey and Wickens (1982,p.86), it is not valid to apply the standard form of the  $F$  statistic to (6) in testing  $H$  in (7). For example, Godfrey and Wickens have shown that, for testing  $\phi = 0$ , the usual regression formula omits the factor 2 arising from the asymptotic distribution of  $n^{1/2} \sum_t u_{0t}(\tilde{u}_{0t} v_{lt})$ . Consider now the  $F$  statistic for testing  $c_1 = 0$ . The asymptotic variance of  $n^{-1/2} \sum_t u_{0t} r_t$  is the same as that of

$$n^{-1/2} \sum_t (u_{0t}^4 - 3u_{0t}^2 \sigma_0^2) / (4\sigma_0^2).$$

Under  $H$ ,  $u_{0t} = \epsilon_{0t} \sim \mathcal{NID}(0, \sigma_0^2)$  for all  $t$ . Therefore, the above variance can also be expressed as

$$\begin{aligned} & [E(u_{0t}^8) - (E(u_{0t}^4))^2 + 9\sigma_0^4(E(u_{0t}^4) - E(u_{0t})^2) \\ & \quad - 6\sigma_0^2(E(u_{0t}^6) - E(u_{0t}^4)E(u_{0t}^2))]/(16\sigma_0^4) \\ & = [(105\sigma_0^8 - 9\sigma_0^8) + 9\sigma_0^4(3\sigma_0^4 - \sigma_0^4) - 6\sigma_0^2(15\sigma_0^6 - 3\sigma_0^6)]/(16\sigma_0^4) \\ & = 21\sigma_0^4/8. \end{aligned}$$

However, the limiting value of the regression formula is

$$\sigma_0^2 \text{plim } n^{-1} \sum_t (u_{0t}^3 - 3u_{0t}\sigma_0^2)^2/(16\sigma_0^4) = 3\sigma_0^4/8,$$

which is one-seventh of the correct asymptotic variance.

Denote the standard  $F$  statistics for testing

$$(\rho_0 = 0, \alpha = 0), \phi = 0 \quad \text{and} \quad c_1 = 0$$

by  $F_1$ ,  $F_2$  and  $F_3$ , respectively. Note that the regressors corresponding to the above parameters are asymptotically uncorrelated with each other, and also the regressors with coefficients  $\phi$  and  $c_1$  are asymptotically orthogonal to the regressors of the null model. Therefore, the conditions for the decomposition of the joint test are satisfied [see Godfrey (1988, p.79)]. It follows that, under  $H_0$ ,

$$(p_0 + 1)F_1 + \frac{1}{2}q_0F_2 + \frac{1}{7}F_3 \xrightarrow{\mathcal{D}} \chi^2(p_0 + q_0 + 2).$$

This test statistic will test the standard linear model (1) with normal and spherical disturbances against a broader alternative of non-spherical and non-normal disturbances, as well as against a non-nested alternative. Depending on the situation, it is possible to specialize the test statistic to particular alternatives by retaining the appropriate regressors in equation (6). For example, to test the null model  $H_0$  against a non-nested alternative  $H_1$  and



the possible presence of heteroskedasticity, it would be necessary to test  $\phi = 0$  and  $\alpha = 0$  in the auxiliary regression given by

$$y_t = x_t' \beta + \tilde{u}_{0t} \sum_{l=1}^{q_0} \phi_l v_{lt} + \alpha \hat{y}_{1t} + \epsilon_{0t}.$$

This auxiliary regression equation is simply a specialization of equation (6) with  $\rho_{0j} = 0$  ( $j = 1, 2, \dots, p_0$ ) and  $c_1 = 0$ .

#### 4. MONTE CARLO RESULTS

As mentioned in Section 2, extensive Monte Carlo results have been reported in the literature concerning the violation of the classical conditions in testing both nested and non-nested hypotheses. However, in the econometrics literature these two types of hypothesis testing have been studied separately. Here we shall concentrate on a particular departure from the classical conditions, i.e., serial dependence ( $\bar{I}$ ) (first-order serial correlation), *together* with the non-nested alternative. This specialized form of the joint test is easily obtained by setting  $\rho_{0j} = 0$  ( $j = 2, 3, \dots, p_0$ ),  $\phi_l = 0$  ( $l = 1, 2, \dots, q_0$ ) and  $c_1 = 0$  in equation (6) to give the auxiliary regression

$$y_t = x_t' \beta + \rho_0 \tilde{u}_{0t-1} + \alpha \hat{y}_{1t} + \epsilon_{0t}.$$

The joint hypothesis of  $H : \rho_0 = 0, \alpha = 0$  can be tested by computing the standard  $F$  statistic for  $H$  and then noting  $2F \xrightarrow{D} \chi^2(2)$  under the null hypothesis.

For comparison with our joint test we also consider the J test of Davidson and MacKinnon (1981), which involves testing  $\alpha = 0$  in the auxiliary regression given by

$$y_t = x_t' \beta + \alpha \hat{y}_{1t} + \epsilon_{0t}.$$

The J test can be carried out by the standard  $t$  test. We, however, performed this procedure using a  $\chi^2(1)$  test.

The data were generated by the following non-nested linear regression models with AR(1) disturbances:

$$H_0 : y_{0t} = \sum_{j=1}^4 x_{tj} \beta_j + u_{0t} \quad (8)$$

and

$$H_1 : y_{1t} = \sum_{j=1}^4 z_{tj} \gamma_j + u_{1t} \quad (9)$$

with

$$u_{it} = \rho_i u_{it-1} + \epsilon_{it}, \quad i = 0, 1$$

where we set  $x_{t1} = 1$  and generated  $x_{t2}$  from  $\mathcal{N}(10, 25)$ ,  $x_{t3}$  from the uniform distribution in the interval  $[7.5, 12.5]$  and  $x_{t4}$  from  $\chi^2_{10}$ . To generate the  $z_{tj}$  values, we set  $z_{tj} = x_{tj}$ ,  $j = 1, 2, 3$  and obtained  $z_{t4}$  from  $\mathcal{N}(10, 20)$ . These values of the design matrices,  $X$  and  $Z$ , were kept fixed from one replication to another. Serially correlated disturbances were generated from the AR(1) process by setting  $\rho_0 = \rho_1 = .3$  and  $\rho_0 = \rho_1 = .4$ .  $\epsilon_{it}$ 's ( $i=0,1$ ) were obtained from  $\mathcal{N}(0, 25)$ .

In every experiment, for a given sample size  $n$ , we calculated the above  $\chi^2(1)$  and  $\chi^2(2)$  statistics under the two non-nested models,  $H_0$  and  $H_1$ , both with AR(1) disturbances. The experiments were performed for sample sizes  $n = 40, 50, 70$  and  $100$ , and for each  $n$ , we carried out 500 replications. The results are presented in Tables 2 and 3 using the nominal level of .05. The reported numbers in the tables are proportion of rejections of the model  $y_t = x'_t \beta + u_t$  which has no serial correlation when the true model is either  $H_0$  or  $H_1$  as given in equations (8) and (9).

TABLE 2. Proportion of rejections when  $\rho_0 = \rho_1 = .3$ 

| DGP       | $H_0$       | $H_0$       | $H_1$       | $H_1$       |
|-----------|-------------|-------------|-------------|-------------|
|           | $\chi^2(1)$ | $\chi^2(2)$ | $\chi^2(1)$ | $\chi^2(2)$ |
| $n = 40$  | .048        | .288        | .632        | .688        |
| $n = 50$  | .060        | .370        | .780        | .820        |
| $n = 70$  | .098        | .544        | .828        | .918        |
| $n = 100$ | .222        | .738        | .948        | .982        |

TABLE 3. Proportion of rejections when  $\rho_0 = \rho_1 = .4$ 

| DGP       | $H_0$       | $H_0$       | $H_1$       | $H_1$       |
|-----------|-------------|-------------|-------------|-------------|
|           | $\chi^2(1)$ | $\chi^2(2)$ | $\chi^2(1)$ | $\chi^2(2)$ |
| $n = 40$  | .044        | .494        | .700        | .844        |
| $n = 50$  | .074        | .612        | .658        | .904        |
| $n = 70$  | .112        | .782        | .648        | .942        |
| $n = 100$ | .200        | .932        | .892        | .998        |

When the DGP is given by the null model  $H_0$  with serially correlated disturbances, the estimated proportions for the  $\chi^2(1)$  test require careful interpretation. The situation is analogous to the completely misspecified cases discussed in Section 2 in that the  $\chi^2(1)$  test is designed to detect non-nested component in the design matrix when the design matrix is correctly specified but serial correlation ( $\bar{I}$ ) is present. If we view the  $\chi^2(1)$  test as a non-nested test, then we find that the sizes of the test are too high especially for sample sizes  $n = 70$  and 100. If we regard this as a significance test, then the test is not powerful enough. Our  $\chi^2(2)$  is a two directional test in the directions of non-nested alternatives and serial correlation. When the DGP is  $H_0$ ,  $\chi^2(2)$  does pick up serial correlation with reasonable power. However, there is some loss of power due to ‘overtesting’, since the  $\chi^2(2)$  joint test

seeks to detect both alternative mean specification and  $\bar{I}$ , when only  $\bar{I}$  is present.

Turning to the cases where the data are generated under the non-nested alternative  $H_1$  as in equation (9), it is again observed that serial dependence ( $\bar{I}$ ) has adverse effect on the power of the  $\chi^2(1)$  test. As clearly seen from Table 2 the  $\chi^2(1)$  test is less powerful, for all  $n$ , than the  $\chi^2(2)$  test under the same DGP. This is the problem of ‘undertesting’ which is common to one-directional tests as, in our cases, the  $\chi^2(1)$  test can only detect the non-nested part but not  $\bar{I}$ . Finally, we observe that the joint  $\chi^2(2)$  test has very good power under this setup because this is the situation for which the statistic is designed. And in this situation we observe substantial power loss if the  $\chi^2(1)$  test is used in place of our  $\chi^2(2)$  test.

The results in Table 3 are for  $\rho_0 = \rho_1 = .4$ . Under the DGP  $H_0$ , the  $\chi^2(1)$  test has similar rejection proportions as in Table 2. However, under DGP  $H_1$ , it has overall lower power compared to  $\rho_0 = \rho_1 = .3$  case. This is due to the fact that the  $\chi^2(1)$  test does not take account of serial correlation at all and  $\rho_0 = \rho_1 = .4$  has more adverse effect than  $\rho_0 = \rho_1 = .3$ . The attractiveness of  $\chi^2(2)$  test is also clearly revealed from Table 3 results. Under DGP  $H_0$ , the  $\chi^2(2)$  test detects serial correlation 50 percent of the time even for sample size 40. And when the departure is two-directional, i.e., under DGP  $H_1$ , the powers of the  $\chi^2(2)$  test are very close to one even for small sample sizes. This limited Monte Carlo study could easily be extended in various directions. However, what we presented is very much indicative of the general finite sample behaviour of our proposed joint test.

## 5. CONCLUSION

In this paper we have presented some simple joint tests of non-nested models and general error specifications. The joint tests for non-nested specifications and for one or more departures from the classical conditions of serial independence, homoskedasticity and normality were developed within

the context of locally equivalent alternatives. These tests represent a simple asymptotic solution to the “pre-testing” problem as applied to non-nested linear regression models. Results of our limited Monte Carlo study demonstrate that the proposed tests have good finite sample properties. If the null hypothesis is not rejected by the joint tests, standard regression analysis would follow for the underlying null model. However, if the null is rejected, it is not possible to infer whether it is rejected because of the non-nested alternative or through departures from the classical conditions regarding the disturbances.

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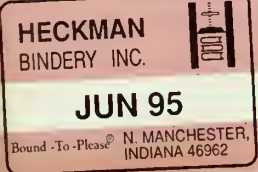












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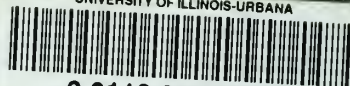


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